1 Relations and Functions

Fastrack Revision

- Let A and B be two non-empty sets. Then a relation R from A to B is a subset of $A \times B$. i.e., R is a relation from A to B $\Leftrightarrow R \subseteq (A \times B)$.
- In set X, the relation R given by $R = \phi \subseteq X \times X$ is called an empty relation if no element of X is related to any element of X.
- In set X, the relation R given by $R = (X \times X) \subseteq (X \times X)$ is called a universal relation if each element of X is related to every element of X.
- If A and B are two non-empty sets and R is a relation from A to B, such that $R = \{(a, b) : a \in A, b \in B\}$ then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$.
- A relation R in set X for which $(a, a) \in R \, \forall a \in X$, is reflexive relation.
- A relation R in set X which satisfies the condition that $(a, b) \in R$ implies $(b, a) \in R$ for any $a, b \in X$, is symmetric relation.
- A relation R is said to be anti-symmetric relation, iff

  $$(a, b) \in R \text{ and } (b, a) \in R$$
  $$\Rightarrow \quad a = b \text{ for any } a, b \in X$$

  In case $a \neq b$, then even if $(a, b) \in R$ and $(b, a) \in R$ holds, the relation cannot be anti-symmetric.
- A relation R in set X satisfying the condition $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for any $a, b, c \in X$ is transitive relation.
- A relation R in set X, which is reflexive, symmetric and transitive, is said to be an equivalence relation.
- Equivalence class $[a]$ containing $a \in X$ for an equivalence relation R in set X is the subset of set X containing all elements related to a.
- Let A and B be two non-empty sets. Then a relation f from A to B is function, if every element in A has a unique image in B and which is denoted by $f : A \to B$.
- A function $f : X \to Y$ is one-one (or injective) if

  $$f(x_1) = f(x_2) \quad \Rightarrow \quad x_1 = x_2, \forall x_1, x_2 \in X.$$
- A function $f : X \to Y$ is onto (or surjective) if given any $y \in Y, \exists \, x \in X$, such that $f(x) = y$.
- A function $f : X \to Y$ is bijective if f is both one-one and onto.
- Given a finite set X, a function $f : X \to Y$ is one-one (respectively onto) if and only if f is onto (respectively one-one). This is the characteristic property of a finite set. This is not true for infinite set.

Practice Exercise

Multiple Choice Questions

Q 1. Let $A = \{3, 5\}$. Then number of reflexive relation on A is: *(CBSE 2023)*

a. 2 b. 4 c. 0 d. 8

Q 2. Let R be a relation in the set N given by $R = \{(a, b) : a = b - 2, b > 6\}$, then: *(CBSE 2023)*

a. $(8, 7) \in R$ b. $(6, 8) \in R$
c. $(3, 8) \in R$ d. $(2, 4) \in R$

Q 3. If $R = \{(x, y); x, y \in Z, x^2 + y^2 \leq 4\}$ is a relation in set Z, then domain of R is: *(CBSE 2021 Term-1)*

a. $\{0, 1, 2\}$ b. $\{-2, -1, 0, 1, 2\}$
c. $\{0, -1, -2\}$ d. $\{-1, 0, 1\}$

Q 4. A relation R in set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (1, 2), (2, 2), (3, 3)\}$. Which of the following ordered pair in R shall be removed to make it an equivalence relation in A? *(CBSE SQP 2021 Term-1)*

a. (1, 1) b. (1, 2) c. (2, 2) d. (3, 3)

Q 5. Let set $X = \{1, 2, 3\}$ and a relation R is defined in X as $R = \{(1, 3), (2, 2), (3, 2)\}$, then minimum ordered pairs which should be added in relation R to make it reflexive and symmetric, are: *(CBSE 2021 Term-1)*

a. $\{(1, 1), (2, 3), (1, 2)\}$ b. $\{(3, 3), (3, 1), (1, 2)\}$
c. $\{(1, 1), (3, 3), (3, 1), (2, 3)\}$ d. $\{(1, 1), (3, 3), (3, 1), (1, 2)\}$

Q 6. Let $A = \{x, y, z\}$ and $B = \{1, 2\}$, then the number of relations from A to B is:

a. 32 b. 64 c. 128 d. 8

Q 7. The relation R defined in $A = \{1, 2, 3\}$ by aRb, if $|a^2 - b^2| \leq 5$. Which of the following is false?

a. $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)\}$
b. $R^{-1} = R$
c. Domain of $R = \{1, 2, 3\}$
d. Range of $R = \{5\}$

Q 8. A relation in a set A is called .... relation, if each element of A is related to itself.

a. reflexive b. symmetric
c. transitive d. None of these

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Q 9. A relation R on a set A is called .... if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, then $a_1, a_2, a_3 \in A$.

a. reflexive

b. symmetric

c. transitive

d. None of the above

Q 10. Total number of equivalence relations defined in the set $S = \{a, b, c\}$ is:

a. 5 b. 31 c. 23 d. 33

Q 11. Let $A = \{1, 2, 3\}$. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is:

a. 1 b. 2 c. 3 d. 4

Q 12. Let $A = \{1, 2, 3\}$. Then number of equivalence relations containing (1, 2) is:

a. 1 b. 2 c. 3 d. 4

Q 13. Let R be the relation in the set $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Choose the correct answer.

a. R is reflexive and symmetric but not transitive.

b. R is reflexive and transitive but not symmetric.

c. R is symmetric and transitive but not reflexive.

d. R is an equivalence relation.

Q 14. Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 3)\}$ be a relation in A. Then, the minimum number of ordered pairs may be added, so that R becomes an equivalence relation, is:

a. 7 b. 5 c. 1 d. 4

Q 15. Let the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$, given by $R = \{(a, b) : |a - b|$ is a multiple of $4\}$. Then [1], the equivalence class containing 1, is:

a. {1, 5, 9} b. {0, 1, 2, 5}

c. ϕ d. A

Q 16. Let S be the set of all real numbers. Then, the relation $R = \{(a, b) : 1 + ab > 0\}$ on S is:

a. reflexive and symmetric but not transitive

b. reflexive and transitive but not symmetric

c. symmetric and transitive but not reflexive

d. reflexive, transitive and symmetric

Q 17. If R and R' are symmetric relations (not disjoint) on a set A, then the relation $R \cap R'$ is:

a. reflexive b. symmetric

c. transitive d. None of these

Q 18. Let R be a relation on the set N of natural numbers denoted by $nRm \Leftrightarrow n$ is a factor of m ($i.e., n \mid m$). Then, R is:

a. reflexive and symmetric

b. transitive and symmetric

c. equivalence

d. reflexive, transitive but not symmetric

Q 19. Let n be a fixed positive integer. Let a relation R be defined in I (the set of all integers) as follows: aRb iff $n \mid (a - b)$, that is, iff $a - b$ is divisible by n. Then, the relation R is:

a. reflexive only b. symmetric only

c. transitive only d. an equivalence relation

Q 20. For real numbers x and y, we write $xRy \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then, the relation R is:

a. reflexive b. symmetric

c. transitive d. None of these

Q 21. Let T be the set of all triangles in the Euclidean plane and let a relation R on T be defined as aRb if a is congruent to b $\forall a, b \in T$. Then R is:

a. reflexive but not transitive

b. transitive but not symmetric

c. equivalence

d. None of the above

Q 22. Let R be the relation in the set A of all books in a library of a college given by $R = \{(x, y) : x$ and y have same number of pages$\}$. Then, R is:

a. not reflexive b. not symmetric

c. not transitive d. an equivalence relation

Q 23. Let S be the set of all points in a plane and R be a relation on S defined as $R = \{(P, Q) :$ distance between P and Q is less than 2 units$\}$ then, R is:

a. reflexive but not symmetric

b. symmetric and transitive

c. reflexive and transitive

d. reflexive and symmetric but not transitive

Q 24. Let X be a family of sets and R be a relation in X defined by 'A is disjoint from B'. Then, relation R is:

a. reflexive b. symmetric

c. transitive d. anti-symmetric

Q 25. If $n \mid m$ means that n is a factor of m, then relation '|' in $Z - \{0\}$ is:

a. reflexive and symmetric

b. symmetric and transitive

c. reflexive, symmetric and transitive

d. reflexive, transitive and not symmetric

Q 26. Suppose $A = \{1, 2, 3, ..., 9\}$ and R be the relation on $A \times A$ defined by $(a, b) R (c, d)$ if $a + d = b + c$ for all $(a, b), (c, d) \in A \times A$. Then the number of elements in equivalence class related to [(2, 5)] is:

a. 6 b. 5 c. 4 d. 3

Q 27. If $f(x) = |\cos x|$, then $f\left(\frac{3\pi}{4}\right)$ is: (CBSE 2023)

a. 1 b. −1 c. $-\frac{1}{\sqrt{2}}$ d. $\frac{1}{\sqrt{2}}$

Q 28. Let $f : R \to R$ be defined by $f(x) = x^2 + 1$. Then, pre-images of 17 and −3, respectively, are:

a. ϕ, {4, −4} b. {3, −3}, ϕ

c. {4, −4}, ϕ d. {4, −4} {2, −2}

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Q 29. Let $A=\{1, 2, 3\}$, $B=\{4, 5, 6, 7\}$ and let $f=\{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Based on the given information, f is best defined as:
a. surjective function
b. injective function
c. bijective function
d. function

Q 30. A function $f: R \rightarrow R$ defined by $f(x)=2+x^2$ is:
a. not one-one
b. one-one
c. not onto
d. neither one-one nor onto

Q 31. Let $X=\{x^2 : x \in N\}$ and the function $f: N \rightarrow X$ is defined by $f(x)=x^2$, $x \in N$. Then this function is:
a. injective only
b. not bijective
c. surjective only
d. bijective

Q 32. The function $f: R \rightarrow R$ defined as $f(x)=x^3$ is:
a. one-one but not onto
b. not one-one but onto
c. neither one-one nor onto
d. one-one and onto

Q 33. Let $f: N \rightarrow N$ defined by $f(x)=x^2+x+1, x \in N$, then f is:
a. one-one and onto
b. many-one and onto
c. one-one but not onto
d. None of the above

Q 34. Let $f: R \rightarrow R$ be a function defined by $f(x)=\frac{x^2-8}{x^2+2}$, then f is:
a. one-one but not onto
b. one-one and onto
c. onto but not one-one
d. neither one-one nor onto

Q 35. Let $f: R \rightarrow R$ be defined as $f(x)=3x$. Choose the correct answer.
a. f is one-one onto
b. f is many-one onto
c. f is one-one but not onto
d. f is neither one-one nor onto

Q 36. Let $f: R \rightarrow R$ be defined as $f(x)=x^4$. Choose the correct answer.
a. f is one-one onto
b. f is many-one onto
c. f is one-one but not onto
d. f is neither one-one nor onto

Q 37. Let $A=\{x : -1 \leq x \leq 1\}$ and $f: A \rightarrow A$ is a function defined by $f(x)=x|x|$, then f is:
a. a bijection
b. injection but not surjection
c. surjection but not injection
d. neither injection nor surjection

Q 38. Let $f:[0, \infty) \rightarrow [0, 2]$ be defined by $f(x)=\frac{2x}{1+x}$, then f is:
a. one-one but not onto
b. onto but not one-one
c. both one-one and onto
d. neither one-one nor onto

Q 39. Let $A=R-\{3\}$, $B=R-\{1\}$. Let $f: A \rightarrow B$ be defined by $f(x)=\frac{x-2}{x-3}$. Then,
a. f is bijective
b. f is one-one but not onto
c. f is onto but not one-one
d. None of the above

Q 40. A function f from the set of natural numbers to integers is defined by $f(n)=\begin{cases}\frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even}\end{cases}$ is:
a. one-one but not onto
b. onto but not one-one
c. one-one and onto both
d. neither one-one nor onto

Assertion & Reason Type Questions

Directions (Q. Nos. 41-48): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 41. Assertion (A): The relation R in a set $A=\{1, 2, 3, 4\}$ defined by $R=\{(x, y): 3x-y=0\}$ have the Domain $=\{1, 2, 3, 4\}$ and Range $=\{3, 6, 9, 12\}$.
Reason (R): Domain and range of the relation (R) is respectively the set of all first and second entries of the distinct ordered pair of the relation.

Q 42. Assertion (A): If R is a relation defined on the set of natural numbers N such that $R=\{(x, y): x, y \in N$ and $2x+y=24\}$, then R is an equivalence relation.
Reason (R): A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.

Q 43. Assertion (A): If the relation R defined in $A=\{1, 2, 3\}$ by aRb, if $|a^2-b^2| \leq 5$, then $R^{-1}=R$.
Reason (R): For above relation, domain of $R^{-1}=$ Range of R.

Q 44. Assertion (A): A function $y=f(x)$ defined by $x^2-\cot^{-1} y=\pi$, then domain of $f(x)=R$.
Reason (R): $\cot^{-1} y \in (0, \pi)$.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Q 45. Assertion (A): A function $f: R \to R$ satisfies the equation $f(x) - f(y) = x - y \; \forall \; x, y \in R$ and $f(3) = 2$, then $f(xy) = xy - 1$.

Reason (R): $f(x) = f\left(\frac{1}{x}\right) \forall \; x \in R, x \neq 0$ and $f(2) = \frac{7}{3}$ if $f(x) = \frac{x^2 + x + 1}{x^2 - x + 1}$.

Q 46. Assertion (A): The relation R on the set $N \times N$, defined by $(a, b) \, R \, (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$ is an equivalence relation.

Reason (R): Any relation R is an equivalence relation, if it is reflexive, symmetric and transitive.

Q 47. Assertion (A): The relation R given by $R = \{(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)\}$ on a set $A = \{1, 2, 3, 4\}$ is not symmetric.

Reason (R): For symmetric relation, $R = R^{-1}$.

Q 48. Assertion (A): The relation $f: \{1, 2, 3, 4\} \to \{x, y, z, p\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is a bijective function.

Reason (R): The function $f: \{1, 2, 3\} \to \{x, y, z, p\}$ such that $f = \{(1, x), (2, y), (3, z)\}$ is one-one.

(CBSE SQP 2023-24)

Answers

1. (b)	2. (b)	3. (b)	4. (b)	5. (c)	6. (b)	7. (d)	8. (a)	9. (c)	10. (a)
11. (a)	12. (b)	13. (b)	14. (a)	15. (a)	16. (a)	17. (b)	18. (d)	19. (d)	20. (a)
21. (c)	22. (d)	23. (d)	24. (b)	25. (d)	26. (a)	27. (d)	28. (c)	29. (b)	30. (d)
31. (d)	32. (d)	33. (c)	34. (d)	35. (a)	36. (d)	37. (a)	38. (a)	39. (a)	40. (c)
41. (a)	42. (d)	43. (b)	44. (d)	45. (b)	46. (a)	47. (a)	48. (d)		

Case Study Based Questions

Case Study 1

In two different societies, there are some school going students—including girls as well as boys. Satish forms two sets with these students, as his college project.

Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4\}$ where a_i's and b_i's are the school going students of first and second society respectively.

Satish decides to explore these sets for various types of relations and functions.

Based on the above information, solve the following questions:

Q 1. Satish wishes to know the number of reflexive relations defined on set A. How many such relations are possible?

a. 0
b. 2^5
c. 2^{10}
d. 2^{20}

Q 2. Let $R: A \to A$, $R = \{(x, y): x$ and y are students of same sex$\}$. Then relation R is:

a. reflexive only
b. reflexive and symmetric but not transitive
c. reflexive and transitive but not symmetric
d. an equivalence relation

Q 3. Satish and his friend Rajat are interested to know the number of symmetric relations defined on both the sets A and B, separately. Satish decides to find the symmetric relation on set A, while Rajat decides to find the symmetric relation on set B. What is difference between their results?

a. 1024
b. $2^{10}(15)$
c. $2^{10}(31)$
d. $2^{10}(63)$

Q 4. Let $R: A \to B$, $R = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_3, b_3), (a_4, b_2), (a_5, b_2)\}$, then R is:

a. neither one-one nor onto
b. one-one but, not onto
c. only onto, but not one-one
d. not a function

Q 5. To help Satish in his project, Rajat decides to form onto function from set A to B. How many such functions are possible?

a. 342
b. 240
c. 729
d. 1024

Solutions

1. Number of reflexive relations defined on a set of n elements

$$= 2^{n(n-1)}$$

Therefore, number of reflexive relations defined on set A having 5 elements $= 2^{5 \times 4} = 2^{20}$.

So, option (d) is correct.

2. As $(x, x) \in R$ for all $x \in A$ when x is either boy or girl.

So, R is reflexive.

Let $(x, y) \in R$ that is, x and y are of same sex.

That means, y and x are also of same sex.

This implies, $(y, x) \in R$.

So, R is symmetric.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Also let $(x, y) \in R$ and $(y, z) \in R$.

That means, x and y are of same sex; y and z are same sex. Clearly, x and z will also be of same sex. That implies, $(x, z) \in R$.

So, R is transitive.

Therefore, R is equivalence relation.

So, option (d) is correct.

3. No. of Symmetric relations defined on a set of n elements

$$= 2^{\frac{n(n+1)}{2}}.$$

Therefore, number of symmetric relations defined on set A having 5 elements $= 2^{\frac{5 \times 6}{2}} = 2^{15}$

Therefore, number of symmetric relations defined on set B having 4 elements $= 2^{\frac{5 \times 4}{2}} = 2^{10}$

Hence, the required difference is $2^{15} - 2^{10} = 2^{10}(31)$.

So, option (c) is correct.

4. For the element $a_1 \in A$, we have different images under R.

Note that, we have $(a_1, b_1), (a_1, b_2) \in R$.

So, R is not a function.

So, option (d) is correct.

5. If A and B are two sets having m and n elements respectively such that $m \geq n$, then total number of onto functions from set A to set B is

$$= \sum_{r=0}^{n} (-1)^r \times {}^nC_r \times (n-r)^m.$$

Here, $n(A) = 5$ i.e., $m = 5$ and $n(B) = 4$ i.e., $n = 4$

So, the number of onto functions from set A to set B

$$= \sum_{r=0}^{4} (-1)^r \times {}^4C_r \times (4-r)^5$$

$$= (-1)^0 \times {}^4C_0 \times (4-0)^5 + (-1)^1 \times {}^4C_1 \times (4-1)^5 + (-1)^2 \times {}^4C_2 \times (4-2)^5 + (-1)^3 \times {}^4C_3 \times (4-3)^5 + (-1)^4 \times {}^4C_4 \times (4-4)^5$$

$$= 1 \times 1 \times (4)^5 + (-1) \times 4 \times (3)^5 + 1 \times 6 \times (2)^5 + (-1) \times 4 \times 1 + 1 \times 1 \times 0$$

$$= 1024 - 972 + 192 - 4 = 240.$$

So, option (b) is correct.

Case Study 2

Sherlin and Danju are playing Ludo at home during Covid-19. While rolling the dice, Sherlin's sister Raji observed and noted the possible outcomes of the throw every time belongs to set $\{1, 2, 3, 4, 5, 6\}$. Let A be the set of players while B be the set of all possible outcomes. i.e, $A = \{S, D\}, B = \{1,2,3,4,5,6\}$

Based on the given information, solve the following questions:

Q 1. Let $R: B \to B$ be defined by $R = \{(x, y) : y$ is divisible by $x\}$ is:

a. reflexive and transitive but not symmetric

b. reflexive and symmetric and not transitive

c. not reflexive but symmetric and transitive

d. equivalence

Q 2. Raji wants to know the number of functions from A to B. How many number of functions are possible?

a. 6^2 b. 2^6 c. $6!$ d. 2^{12}

Q 3. Let R be a relation on B defined by $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$. Then R is:

a. symmetric b. reflexive

c. transitive d. None of these

Q 4. Raji wants to know the number of relations possible from A to B. How many numbers of relations are possible?

a. 6^2 b. 2^6 c. $6!$ d. 2^{12}

Q 5. Let $R: B \to B$ be defined by $R = \{(1, 1), (1, 2), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$, then R is:

a. symmetric

b. reflexive and transitive

c. transitive and symmetric

d. equivalence

Solutions

1. $\therefore R = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 4), (2, 6), (3, 6)\}$

For reflexive, we know that x is divisible by x for all $x \in B$.

$\therefore (x, x) \in R$ for all $x \in R$. So, R is reflexive.

For symmetry, we observe that 6 is divisible by 2. This means that $(2, 6) \in R$ but $(6, 2) \notin R$. So, R is not symmetric.

For transitivity, let $(x, y) \in R$ and $(y, z) \in R$, then z is divisible by x.

$\Rightarrow \quad (x, z) \in R$

For example, 4 is divisible by 2, 2 is divisible by 1

So, 4 is divisible by 1. So, R is transitive.

So, option (a) is correct.

2. Here, $n(A) = 2$ and $n(B) = 6$

$\therefore$ Number of functions from A to $B = [n(B)]^{n(A)} = 6^2$.

So, option (a) is correct.

3. Here $B = \{1, 2, 3, 4, 5, 6\}$ and $R = \{(1, 2), (2, 2), (1, 3), (3, 4), (3, 1), (4, 3), (5, 5)\}$

Since, $(a, a) \notin R$, for every $a \in \{1, 2, 3, 4, 5, 6\}$

Therefore, R is not reflexive.

Now, since, $(1, 2) \in R$ but $(2, 1) \notin R$.

Therefore, R is not symmetric.

Also, it is observed that $(a, b), (b, c) \in R$

$\Rightarrow (a, c) \notin R$, for any $a, b, c \in \{1, 2, 3, 4, 5, 6\}$

As $(1, 3), (3, 4) \in R$, but $(1, 4) \notin R$

Therefore R is not transitive.

So, option (d) is correct.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

4. Here, $n(A) = 2$ and $n(B) = 6$

$\therefore$ Number of relations from A to $B = 2^{n(A) \times n(B)}$

$= 2^{2\times6} = 2^{12}$.

So, option (d) is correct.

5. Here, $B = \{1, 2, 3, 4, 5, 6\}$

and $R = \{(1, 2), (2, 2), (1, 1), (3, 3), (4, 4), (5, 5), (6, 6)\}$

Since, $(a, a) \in R$ for every $a \in \{1, 2, 3, 4, 5, 6\}$

So, R is reflexive.

Now, since $(1, 2) \in R$ but $(2, 1) \notin R$.

So, R is not symmetric.

Also, it is observed that $(a, b), (b, c) \in R$

$\Rightarrow$ $(a, c) \in R$ for any $a, b, c \in \{1, 2, 3, 4, 5, 6\}$

So, R is transitive.

So, option (b) is correct.

Case Study 3

Archana visited the exhibition along with her family. The exhibition had a huge swing, which attracted many children. Archana found that the swing traced the path of a parabola as given by $y = x^2$.

Based on the above information, solve the following questions:

Q 1. Let $f : R \to R$ be defined by $f(x) = x^2$ is ....

a. neither surjective nor injective

b. surjective

c. injective

d. bijective

Q 2. Let $f : N \to N$ be defined by $f(x) = x^2$ is ....

a. surjective but not injective

b. surjective

c. injective

d. bijective

Q 3. Let $f : \{1, 2, 3, ...\} \to \{1, 4, 9, ...\}$ be defined by $f(x) = x^2$ is ....

a. bijective

b. surjective but not injective

c. injective but surjective

d. Neither surjective nor injective

Q 4. Let $N \to R$ be defined by $f(x) = x^2$. Range of the function among the following is ....

a. {1, 4, 9, 16, ...}   b. {1, 4, 8, 9, 10, ...}

c. {1, 4, 9, 15, 16, ...}   d. {1, 4, 8, 16, ...}

Q 5. The function $f : Z \to Z$ defined by $f(x) = x^2$ is ....

a. neither injective nor surjective

b. injective

c. surjective

d. bijective

Solutions

1. $f : R \to R$ is given by $f(x) = x^2$.

It is seen that $f(-1) = f(1) = 1$ but $-1 \neq 1$

So, f is not injective.

Now, $-2 \in R$. But there does not exist any element $x \in R$ such that $f(x) = x^2 = -2$.

So, f is not surjective.

Hence, function is neither surjective nor injective.

So, option (a) is correct.

2. $f : N \to N$ is given by $f(x) = x^2$

It is seen that for $x, y \in N$, $f(x) = f(y)$

$\Rightarrow$ $x^2 = y^2$

$\Rightarrow$ $x = y$

[$\because$ x and y are positive numbers]

So, f is injective.

Now, $2 \in N$ but there does not exist any x in N such that $f(x) = x^2 = 2$.

It means there is some element in co-domain in which do not have any images. Therefore, f is not surjective.

So, option (c) is correct.

3. $f : \{1, 2, 3, ...\} \to \{1, 4, 9, ...\}$ is given by

$f(x) = x^2$.

It is seen that for $x_1, x_2 \in \{1, 2, 3, ...\}$

$f(x_1) = f(x_2)$

$\Rightarrow$ $x_1^2 = x_2^2$

$\Rightarrow$ $x_1 = x_2$

[$\because$ x_1 and x_2 are positive numbers]

So, f is injective.

Now, there exist any element x in $\{1, 2, 3, ...\}$ such $f(x) = x^2$

e.g. At $x = 1$, $f(1) = 1$

At $x = 2$, $f(2) = 4$

At $x = 3$, $f(3) = 9$ ...

TR!CK

A function is an onto (surjective) function, if its range is equal to co-domain.

It means all elements in co-domain have images.

So, f is surjective.

Hence, f is bijective function.

So, option (a) is correct.

4. $f : N \to R$ is given by $f(x) = x^2$

At $x = 1$, $f(1) = 1$

At $x = 2$, $f(2) = 4$

At $x = 3$, $f(3) = 9$ ......

So, range of $f(x) = \{1, 4, 9, 16, ...\}$

So, option (a) is correct.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

5. $f: Z \to Z$ is given by $f(x) = x^2$

It is seen that $f(-1) = f(1) = 1$

but $-1 \neq 1$.

So, f is not injective.

Now, $-2 \in Z$. But there does not exist any elements $x \in Z$ such that $f(x) = x^2 = -2$

So, f is not surjective.

Hence, function f is neither injective nor surjective.

So, option (a) is correct.

Case Study 4

Consider the mapping $f: A \to B$ is defined by $f(x) = \dfrac{x-1}{x-2}$ such that f is a bijection.

Based on the above information, solve the following questions:

Q 1. Domain of f is:

a. $R - \{2\}$ b. R c. $R - \{1, 2\}$ d. $R - \{0\}$

Q 2. Range of f is:

a. R b. $R - \{1\}$ c. $R - \{0\}$ d. $R - \{1, 2\}$

Q 3. If $g: R - \{2\} \to R - \{1\}$ is defined by $g(x) = 2f(x) - 1$, then $g(x)$ in terms of x is:

a. $\dfrac{x+2}{x}$ b. $\dfrac{x+1}{x-2}$ c. $\dfrac{x-2}{x}$ d. $\dfrac{x}{x-2}$

Q 4. The function g defined above, is:

a. one-one b. many-one

c. into d. None of these

Q 5. A function $f(x)$ is said to be one-one iff:

a. $f(x_1) = f(x_2) \Rightarrow -x_1 = x_2$

b. $f(-x_1) = f(-x_2) \Rightarrow -x_1 = x_2$

c. $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

d. None of the above

Solutions

1. For $f(x)$ to be defined $x - 2 \neq 0$ i.e., $x \neq 2$

$\therefore$ Domain of $f = R - \{2\}$

So, option (a) is correct.

2. Let $y = f(x)$, then $y = \dfrac{x-1}{x-2}$

$\Rightarrow xy - 2y = x - 1$

$\Rightarrow xy - x = 2y - 1$

$\Rightarrow x = \dfrac{2y-1}{y-1}$

Since, $x \in R - \{2\}$

therefore $y \neq 1$

Hence, range of $f = R - \{1\}$

So, option (b) is correct.

3. We have, $g(x) = 2f(x) - 1$

$$= 2\left(\frac{x-1}{x-2}\right) - 1$$

$$= \frac{2x - 2 - x + 2}{x-2} = \frac{x}{x-2}$$

So, option (d) is correct.

4. We have, $g(x) = \dfrac{x}{x-2}$

Let $g(x_1) = g(x_2)$

$\Rightarrow \dfrac{x_1}{x_1 - 2} = \dfrac{x_2}{x_2 - 2}$

$\Rightarrow x_1x_2 - 2x_1 = x_1x_2 - 2x_2$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

Thus, $g(x_1) = g(x_2)$

$\Rightarrow x_1 = x_2$

Hence, $g(x)$ is one-one.

Also, range of $g(x)$ = co-domain

So, $g(x)$ is onto.

So, option (a) is correct.

5. $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

So, option (c) is correct.

Case Study 5

Students of Grade 12, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line $y = x - 4$. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

Based on the above information, solve the following questions:

Q 1. Let relation R be defined by $R = \{(L_1, L_2): L_1 \parallel L_2$ where $L_1, L_2 \in L\}$ then show that the relation R is an equivalence relation.

Q 2. Let $R = \{(L_1, L_2): L_1 \perp L_2$ where $L_1, L_2 \in L\}$, then show that R is only symmetric relation.

Q 3. Show that the function $f: R \to R$ defined by $f(x) = x - 4$ is bijective.

Solutions

1. Here, $R = \{(L_1, L_2): L_1 \parallel L_2$ where $L_1, L_2 \in L\}$

R is reflexive as any line L_1 is parallel to itself

i.e., $(L_1, L_1) \in R$

Now, let $(L_1, L_2) \in R$

$\Rightarrow L_1$ is parallel to $L_2 \Rightarrow L_2$ is parallel to L_1

$\Rightarrow (L_2, L_1) \in R$

So, R is symmetric.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Now, let $(L_1, L_2), (L_2, L_3) \in R$

$\Rightarrow$ L_1 is parallel to L_2, also L_2 is parallel to L_3

$\Rightarrow$ L_1 is parallel to L_3

$\Rightarrow$ $(L_1, L_3) \in R$. So, R is transitive.

Hence, R is an equivalence relation.

2. Here, $R = \{(L_1, L_2): L_1 \perp L_2 \text{ where } L_1, L_2 \in L\}$

R is not reflexive as any line L_1 is not perpendicular to itself.

i.e., $(L_1, L_1) \notin R$

Now, let $(L_1, L_2) \in R$

$\Rightarrow$ L_1 is perpendicular to L_2

$\Rightarrow$ L_2 is perpendicular to L_1

$\Rightarrow$ $(L_2, L_1) \in R$

So, R is symmetric.

Now, let $(L_1, L_2), (L_2, L_3) \in R$

$\Rightarrow$ L_1 is perpendicular L_2, also L_2 is perpendicular to L_3

$\Rightarrow$ L_1 is parallel to L_3 i.e., L_1 is not perpendicular to L_3.

$\Rightarrow$ $(L_1, L_3) \notin R$. So, R is not transitive.

3. Here, $f : R \to R$ is defined by $f(x) = x - 4$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 - 4 = x_2 - 4$

$\Rightarrow \quad x_1 = x_2$

Therefore, f is one-one.

For any real number y in R, there exist $(y + 4)$ in R such that

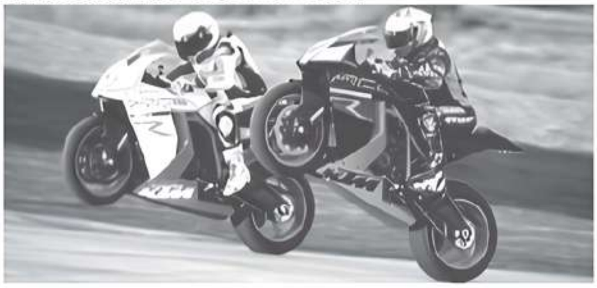
$$f(y + 4) = (y + 4) - 4 = y$$

So, f is onto. Hence, f is bijective.

Case Study 6

An organization conducted bike race under two different categories—Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, solve the following questions: **(CBSE 2023)**

Q 1. How many relations are possible from B to G?

Q 2. Among all the possible relations from B to G, how many functions can be formed from B to G?

Q 3. Let $R: B \to B$ be defined by $R = \{(x, y): x \text{ and } y \text{ are students of the same sex}\}$. Check whether R is an equivalence relation.

Or

A function $f: B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check whether f is bijective. Justify your answer.

Solutions

1. The number of relations from B to G is

$$2^{n(B) \times n(G)} = 2^{3 \times 2} = 2^6 = 64$$

2. The number of functions from B to G is $[n(G)]^{n(B)}$ i.e., 2^3 or 8.

3. **Reflexive**

Since x and x are of the same sex.

So, $(x, x) \in R$ for all x.

$\therefore$ R is reflexive.

Symmetric

If x and y are of the same sex. Then y and x are of the same sex.

i.e. $(x, y) \in R \Rightarrow (y, x) \in R \ \forall x, y$

So, R is symmetric.

Transitive

If x and y are of the same sex; y and z are of the same sex, then x and z are of the same sex.

i.e. $(x, y) \in R, (y, z) \in R \Rightarrow (x, z) \in R \ \forall \ x, y$ and z.

So, R is transitive

Hence, R is an equivalence relation.

Or

Given function $f: B \to G$ such that

$f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$

Here we see that b_1 and b_3 have same image g_1, so it is not one to one function.

Thus, $f(x)$ is not bijective function.

Case Study 7

A relation R on a set A is said to be an equivalence relation on A iff it is:

Reflexive i.e., $(a, a) \in R \ \forall \ a \in A$.

Symmetric i.e., $(a, b) \in R \Rightarrow (b, a) \in R$ for any $a, b \in A$.

Transitive i.e., $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R$ for any $a, b, c \in A$.

Based on the above information, solve the following questions:

Q 1. If the relation $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$ defined on the set $A = \{1, 2, 3\}$, then show that the relation R is only reflexive.

Q 2. If the relation $R = \{(1, 2), (2, 1), (1, 3), (3, 1)\}$ defined on the set $A = \{1, 2, 3\}$, then show that relation R is only symmetric.

Q 3. If the relation R on the set N of all natural numbers defined as $R = \{(x, y): y = x + 5 \text{ and } x < 4\}$, then show that R is not reflexive as well as symmetric but R is transitive.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Q 4. If the relation R on the set $A = \{1, 2, 3, ..., 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$, then show that R is not reflexive, symmetric and transitive.

Q 5. If the relation R on the set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (1,2), (1,3), (2, 1), (2,2), (2,3), (3, 1), (3,2), (3, 3)\}$, then show that R is an equivalence relation.

Solutions

1. Clearly, $(1, 1), (2, 2), (3, 3), \in R$. So, R is reflexive on A.
Since, $(1, 2) \in R$ but $(2, 1) \notin R$. So, R is not symmetric on A.
Since, $(2, 3), \in R$ and $(3, 1) \in R$ but $(2, 1) \notin R$. So, R is not transitive on A.
2. Since, $(1, 1), (2, 2)$ and $(3, 3)$ are not in R.
So, R is not reflexive on A.
Now, $(1, 2) \in R \Rightarrow (2, 1) \in R$
and $(1, 3) \in R \Rightarrow (3, 1) \in R$.
So, R is symmetric
Clearly, $(1, 2) \in R$ and $(2, 1) \in R$ but $(1, 1) \notin R$.
So, R is not transitive on A.
3. We have, $R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$,
where $x, y \in N$.
$\therefore \quad R = \{(1, 6), (2, 7), (3, 8)\}$
Clearly, $(1, 1), (2, 2)$ etc. are not in R. So, R is not reflexive.
Since, $(1, 6) \in R$ but $(6, 1) \notin R$. So, R is not symmetric.
Since, $(1, 6) \in R$ and there is no order pair in R which has 6 as the first element. Same is the case for $(2, 7)$ and $(3, 8)$.
So, R is transitive as transitivity is not contradicted.
4. We have, $R = \{(x, y) : 3x - y = 0\}$,
where $x, y \in A = \{1, 2, ..., 14\}$
$\therefore \quad R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$
Clearly, $(1, 1) \notin R$. So, R is not reflexive on A.
Since, $(1, 3) \in R$ but $(3, 1) \notin R$. So, R is not symmetric on A.
Since, $(1, 3) \in R$ and $(3, 9) \in R$ but $(1, 9) \notin R$. So, R is not transitive on A.
5. Clearly, $(1, 1), (2, 2), (3, 3) \in R$. So, R is reflexive on A.
We find that the ordered pairs obtained by interchanging the components of ordered pairs in R are also in R. So, R is symmetric on A.
For $1, 2, 3 \in A$ such that $(1, 2)$ and $(2, 3)$ are in R implies that $(1, 3)$ is also, in R. So, R is transitive on A.
Thus, R is an equivalence relation.

Very Short Answer Type Questions

Q 1. Let $A = \{0, 1, 2, 3\}$ and a relation R in A is defined as: $R = \{(0, 0), (0, 1), (0, 3), (1, 0), (1, 1), (2, 2), (3, 0), (3, 3)\}$. Is R reflexive, symmetric and transitive? *(NCERT EXEMPLAR)*

Q 2. Show that, in the set of positive integers, the relation 'divide' is reflexive and transitive but not symmetric.

Q 3. A relation R is defined for set $A = \{1, 2, 3\}$ as written below: *(NCERT EXEMPLAR)*
$$R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$$
Write those ordered pairs which when added in R, it becomes the smallest equivalence relation.

Q 4. If $R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$ and $S = \{(2, 3), (3, 2), (2, 2), (3, 3)\}$ are two transitive relations on set $A = \{1, 2, 3\}$ then show that $R \cup S$ is reflexive and symmetric but not transitive.

Q 5. Let X is a set of real numbers then prove that the relation $R = \{(a, b) : a \in X, b \in X \text{ and } a = b\}$ is an equivalence relation. *(NCERT EXERCISE)*

Q 6. Find the number of all one-one functions from the set $A = \{1, 2, 3\}$ to itself. *(NCERT EXERCISE)*

Q 7. Prove that the function $f : R \to R$, given by $f(x) = 2x$, is one-one and onto. *(NCERT EXERCISE)*

Q 8. Prove that $f : N \to N$ defined by $f(x) = x - 1$ and $f(1) = f(2)$, $x > 2$ is onto but not one-one. *(NCERT EXERCISE)*

Q 9. Show that the function $f : R \to R$, defined as $f(x) = x^2$, is neither one-one nor onto.

Q 10. Show that the function $f : [-1, 1] \to R$, given by $f(x) = \dfrac{x}{(x+2)}$ is one-one.

Short Answer Type-I Questions

Q 1. Check if the relation R in the set R of real numbers defined as $R = \{(a, b) : a < b\}$ is (i) symmetric, (ii) transitive. *(CBSE 2020)*

Q 2. Show that the relation R in R defined as $R = \{(a, b) : a \le b\}$, is reflexive and transitive but not symmetric. *(NCERT EXERCISE; CBSE 2019)*

Q 3. Prove that the function $f : N \to N$ defined through $f(x) = 2x$ is one-one but not onto.

Q 4. If $X = \{-1, 1\}$ and the mapping $f : X \to X$ is defined as $f(x) = x^3$, then prove that this mapping is one-one and onto.

Q 5. If the function $f : Q \to Q$ is defined by $f(x) = 3x - 4$ $\forall\ x \in Q$, then show that f is one-one and onto, where Q is the set of rational numbers.

Q 6. Prove that the function f is surjective, where $f: N \to N$ such that
$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$
Is the function injective? Justify your answer. *(CBSE SQP 2022-23)*

Q 7. Show that $f : N \to N$, given by
$$f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$$
is both one-one and onto. *(NCERT EXERCISE)*

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Q 8. Consider a function $f:\left[0,\frac{\pi}{2}\right]\to R$, given by $f(x)=\sin x$ and $g:\left[0,\frac{\pi}{2}\right]\to R$ given by $g(x)=\cos(x)$. Show that f and g are one-one but $f+g$ is not one-one. *(NCERT EXERCISE)*

Q 9. If $R=\{(4,5),(1,4),(4,6),(7,6),(3,7)\}$, then find the value of $R^{-1}OR^{-1}$.

Q 10. Prove that the function $f:N\to N$, defined by $f(x)=x^2+x+1$ is one-one but not onto. *(CBSE 2019)*

Short Answer Type-II Questions

Q 1. Check whether the relation R in R defined by $R=\{(a,b):a\le b^3\}$ is reflexive, symmetric or transitive. *(NCERT EXERCISE)*

Q 2. Prove that in the set of sets, the relation '$\subseteq$' is anti-symmetric.

Q 3. Show that the signum function $f:R\to R$, given by

$$f(x)=\begin{cases}1, & \text{if } x>0\\ 0, & \text{if } x=0\\ -1, & \text{if } x<0\end{cases}$$

is neither one-one nor onto. *(NCERT EXERCISE)*

Q 4. Let the function $f:R\to R$ be defined by $f(x)=\cos x\ \forall, x\in R$, show that f is neither one-one nor onto. *(NCERT EXEMPLAR)*

Q 5. Consider $f:R_+\to[-5,\infty)$ given by $f(x)=9x^2+6x-5$. Show that f is bijective, where R_+ is the set of all non-negative real numbers. *(NCERT EXERCISE; CBSE 2017)*

Long Answer Type Questions

Q 1. Let N be the set of all natural numbers and R be a relation on $N\times N$ defined by $(a,b)R(c,d)\Leftrightarrow ad=bc$ for all $(a,b),(c,d)\in N\times N$. Show that R is an equivalence relation on $N\times N$. Also, find the equivalence class of (2, 6), *i.e.*, [(2, 6)]. *(CBSE SQP 2023-24)*

Or

Define the relation R in the set $N\times N$ as follows:

For $(a,b),(c,d)\in N\times N$, $(a,b)R(c,d)$ iff $ad=bc$. Prove that R is an equivalence relation in $N\times N$. *(CBSE SQP 2022-23)*

Q 2. If N denotes the set of all natural numbers and R is the relation on $N\times N$ defined by $(a,b)R(c,d)$, if $ad(b+c)=bc(a+d)$. Show that R is an equivalence relation. *(CBSE 2023)*

Q 3. If R and S are equivalence relations in a set A, show that $R\cap S$ is also an equivalence relation. *(NCERT EXERCISE)*

Q 4. Given a non-empty set X, define the relation R in $P(X)$ as follows:

For $A,B\in P(X)$, $(A,B)\in R$ iff $A\subset B$. Prove that R is reflexive, transitive and not symmetric. *(CBSE SQP 2022-23)*

Q 5. Let $A=\{x\in Z:0\le x\le 12\}$. Show that $R=\{(a,b):a,b\in A,|a-b|$ is divisible by 4$\}$ is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2]. *(CBSE 2018)*

Q 6. Let $A=R-\{3\},B=R-\{1\}$. Let $f:A\to B$ be defined by $f(x)=\frac{x-2}{x-3},\ \forall\ x\in A$. Show that f is bijective. *(NCERT EXERCISE; CBSE 2017)*

Q 7. Let $f:R-\left\{-\frac{4}{3}\right\}\to R$ be a function defined as $f(x)=\frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not. *(CBSE 2023)*

Q 8. Show that the function $f:R\to R$ defined by $f(x)=\frac{x}{x^2+1},\ \forall\ x\in R$ is neither one-one nor onto. *(CBSE 2018)*

Q 9. Show that the function $f:R\to\{x\in R:-1<x<1\}$ defined by $f(x)=\frac{x}{1+|x|}$, $x\in R$ is one-one and onto function. *(CBSE SQP 2023-24)*

Solutions

Very Short Answer Type Questions

1. Here R is reflexive as $(0,0),(1,1),(2,2),(3,3)\in R$ and symmetric as $(0,1)\in R$ then $(1,0)\in R$ also $(0,3)\in R$ then $(3,0)\in R$ but not transitive because $(1,0)\in R$ and $(0,3)\in R$ while $(1,3)\notin R$.

2. Every positive integer divides itself. Therefore, the relation is reflexive.

   Let three positive integers a,b,c are such that a divides b and b divides c, then a will divide c definitely.

   Let two positive integers a and b are such that a divides b, then b cannot divide a. Therefore the relation is not symmetric. **Hence proved.**

3. (3, 1) is the only ordered pair which when added in R, it becomes smallest equivalence relation.

   As $(1,1),(2,2),(3,3)\in R$, so R is reflexive, $(1,3)\in R$ then $(3,1)\in R$, so R is symmetric.

   At last $(1,3)\in R$, $(3,1)\in R\Rightarrow(1,1)\in R$ so R is transitive also. Hence R is an equivalence relation.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

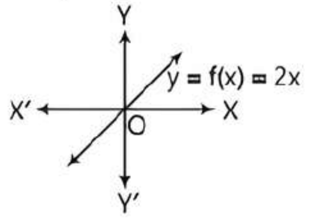
4. Here, $R \cup S = \{(1, 2), (2, 1), (1, 1), (2, 2), (2, 3), (3, 2), (3, 3)\}$
Here, $(1, 1), (2, 2), (3, 3) \in R$ so R is reflexive.
As $(1, 2) \in R \Rightarrow (2, 1) \in R$ and $(2, 3) \in R \Rightarrow (3, 2) \in R$ so, R is symmetric.
Clearly $(1, 2) \in R \cup S$ and $(2, 3) \in R \cup S$
but $(1, 3) \notin R \cup S$
Therefore, the relation $R \cup S$ is not transitive.

Hence proved.

5. Here X = Set of real numbers and
$R = \{(a, b) : a \in X, b \in X \text{ and } a = b\}$
(i) R is reflexive because
$a = a \Rightarrow (a, a) \in R.$
(ii) R is symmetric because if
$(a, b) \in R \Rightarrow a = b$
$\Rightarrow b = a$
$\Rightarrow (b, a) \in R$
(iii) R is transitive because if $(a, b) \in R, (b, c) \in R$
$\Rightarrow a = b, b = c$
$\Rightarrow a = c$
$\Rightarrow (a, c) \in R$
$\therefore$ R is an equivalence relation. **Hence proved.**

6. One-one function from $\{1, 2, 3\}$ to itself is simply a permutation on three symbols 1, 2, 3.
Therefore, total number of one-one maps from $\{1, 2, 3\}$ to itself is same as total number of permutations on three symbols 1, 2, 3 which is $3! = 6$.

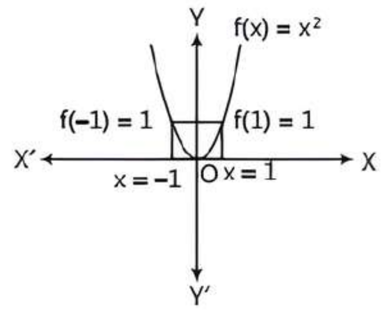
7. f is one-one, as
$f(x_1) = f(x_2)$
$\Rightarrow 2x_1 = 2x_2$
$\Rightarrow x_1 = x_2$



Also, given any real number y in R, there exists $\frac{y}{2}$ in R such that
$$f\left(\frac{y}{2}\right) = 2\left(\frac{y}{2}\right) = y$$
Hence, f is onto. **Hence proved.**

8. Here, $f(1) = f(2)$ but $1 \neq 2$
$\therefore$ f is not one-one. **Hence proved.**
Again, let $f(x) = y$ where $y \in N$
$\Rightarrow x - 1 = y$
$\Rightarrow x = y + 1 \in N \;\forall\; y \in N$
$\therefore$ f is onto. **Hence proved.**

9. Since $f(-1) = 1 = f(1)$, f is not one-one. Also the element (-2) in the co-domain R is not image of any element x in the domain R. Therefore, f is not onto.



The image of 1 and -1 under f is 1.

10. Given : $f : [-1, 1] \rightarrow R$
and $f(x) = \frac{x}{x+2}, x \neq -2$
consider $f(x_1) = f(x_2)$
$\Rightarrow \frac{x_1}{x_1 + 2} = \frac{x_2}{x_2 + 2}$
$\Rightarrow x_1(x_2 + 2) = x_2(x_1 + 2)$
$\Rightarrow x_1x_2 + 2x_1 = x_1x_2 + 2x_2$
$\Rightarrow 2x_1 = 2x_2$
$\therefore x_1 = x_2$
Therefore, f is one-one. **Hence proved.**

Short Answer Type-I Questions

1. Given a relation $R = \{(a, b) : a < b\}$ on R.

TiPS

- *A relation R defined on set A is a symmetric, if $(x, y) \in R$ $\Rightarrow$ $(y, x) \in R$ for any $x, y \in A$*
- *A relation R defined on a set A is a transitive, if $(x, y) \in R$ and $(y, z) \in R$. $\Rightarrow$ $(x, z) \in R$, for any $x, y, z \in A$.*

(i) **Symmetric:** Note that $(2, 3) \in R$ as $2 < 3$
But $(3, 2) \notin R$ as $3 \not< 2$
Hence, the given relation is not symmetric.
(ii) **Transitive:** Let $(a, b) \in R$ and $(b, c) \in R$ be any arbitrary element.
Then, we have $a < b$ and $b < c$
$\Rightarrow a < b < c \Rightarrow a < c$
$\Rightarrow (a, c) \in R$
Hence, the given relation is transitive.

2. Let A = set of real numbers
and $R = \{(a, b) : a \leq b\}$

TiPS

- *A relation R defined an a set A is a reflexive, if $(x, x) \in R \;\forall x \in A$.*
- *A relation R defined on set A is a symmetric, if $(x, y) \in R \Rightarrow (y, x) \in R$ for any $x, y \in A$*
- *A relation R defined on a set A is a transitive, if $(x, y) \in R$ and $(y, z) \in R$. $\Rightarrow$ $(x, z) \in R$, for any $x, y, z \in A$.*

Get More Learning Materials Here : CLICK HERE www.studentbro.in

(i) R is reflexive because $a \le a \Rightarrow a = a$.

(ii) R is not symmetric because a is less than b, but b is not less than a. If 1 is less than 2, then 2 cannot be less than 1.

(iii) R is transitive, because $a \le b, b \le c \Rightarrow a \le c$.

Hence, R is reflexive and transitive but not symmetric. **Hence proved.**

3. Let $x_1, x_2 \in N$ and $f(x_1) = f(x_2)$

$\Rightarrow \quad 2x_1 = 2x_2$

$\Rightarrow \quad x_1 = x_2$

$\therefore$ f is one-one. **Hence proved.**

Let $\quad f(x) = y$ where $y \in N$

$\Rightarrow \quad 2x = y$

$\Rightarrow \quad x = \frac{y}{2} \notin N$

If $y = 5 \in N$ but $x = \frac{5}{2} \notin N$

$\therefore$ f is not onto. **Hence proved.**

4. Here only two elements -1 and 1 are there in set X.

Thus, $\quad f(-1) = (-1)^3 = -1$

and $\quad f(1) = (1)^3 = 1$

i.e., $\quad f(-1) \ne f(1)$

So, f is one-one mapping.

Again, $f(-1) = -1$ and $f(1) = 1$ shows that each element of co-domain has pre-image in domain.

So, f is onto mapping.

Therefore, the mapping f is one-one and onto.

Hence proved.

5. Let x_1 and x_2 are any two rational numbers such that,

$f(x_1) = f(x_2)$ then, $x_1 = x_2$

$\Rightarrow \quad 3x_1 - 4 = 3x_2 - 4$

$\Rightarrow \quad 3x_1 = 3x_2 \quad \Rightarrow \quad x_1 = x_2$

which shows that f is one-one function.

Again let y be any element of co-domain Q such that

$$y = f(x) = 3x - 4$$

then, $\quad 3x = y + 4$

or $\quad x = \frac{y+4}{3}$

Now, put the above value of x in the function

$$f(x) = 3x - 4.$$

$$f\left(\frac{y+4}{3}\right) = 3\left(\frac{y+4}{3}\right) - 4 = (y+4) - 4 = y$$

$\because$ Pre-image $\left(\frac{y+4}{3}\right)$ of each rational number y is also a rational number *i.e.*, in domain.

Hence, function f is onto. **Hence proved.**

6. Here, $f : N \to N$ while

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

(i) When n is odd, *i.e.*, $n = 1$

$$f(1) = \frac{1+1}{2} = \frac{2}{2} = 1$$

When n is even, *i.e.*, $n = 2$

$$f(2) = \frac{2}{2} = 1$$

We see that, f image of 1 and 2 is 1.

$\therefore$ f is not one-one (injective).

(ii) Each member of co-domain is image of one of the member of domain *e.g.*, 1 is the image of numbers 1 and 2.

$\therefore$ f is surjective (onto).

Hence, f is not injective but surjective.

7. Suppose $f(x_1) = f(x_2)$

TR!CK

If x_1 is odd and x_2 is even, then we will have $x_1 + 1 = x_2 - 1$ i.e., $x_2 - x_1 = 2$ which is impossible. Similarly, the possibility of x_1 being even and x_2 being odd can also be ruled out, using the similar argument. Therefore both x_1 and x_2 must be either odd or even.

Suppose both x_1 and x_2 are odd. Then $f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$. Similarly, if both x_1 and x_2 are even, then also $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$. Thus, f is one-one. Also, any odd number $2r + 1$ in the co-domain N is the image of $2r + 2$ in the domain N and any even number $2r$ in the co-domain N is the image of $2r - 1$ in the domain N. Thus, f is onto.

Hence proved.

8. Since for any two distinct elements x_1 and x_2 in $\left[0, \frac{\pi}{2}\right]$, $\sin x_1 \ne \sin x_2$ and $\cos x_1 \ne \cos x_2$, both f and g must be one-one.

But $\quad (f+g)(0) = \sin 0 + \cos 0 = 1$

and $\quad (f+g)\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} + \cos\frac{\pi}{2} = 1.$

Therefore, $f + g$ is not one-one. **Hence proved.**

9. Given that, $R = \{(4, 5), (1, 4), (4, 6), (7, 6), (3, 7)\}$

$\therefore \quad R^{-1} = \{(5, 4), (4, 1), (6, 4), (6, 7), (7, 3)\}$

Now, we see that,

$(5, 4) \in R^{-1}$ and $(4, 1) \in R^{-1} \Rightarrow (5, 1) \in R^{-1}OR^{-1}$

$(6, 4) \in R^{-1}$ and $(4, 1) \in R^{-1} \Rightarrow (6, 1) \in R^{-1}OR^{-1}$

and $(6, 7) \in R^{-1}$ and $(7, 3) \in R^{-1} \Rightarrow (6, 3) \in R^{-1}OR^{-1}$

Hence, $\quad R^{-1}OR^{-1} = \{(5, 1), (6, 1), (6, 3)\}$.

10. Let $x, y \in N$ such that $f(x) = f(y)$

$\Rightarrow \quad x^2 + x + 1 = y^2 + y + 1$

TiP

f is one-one iff $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in X$ or f is one-one iff $f(x_1) \ne f(x_2) \Rightarrow x_1 \ne x_2$ for all $x_1, x_2 \in X$.

$\Rightarrow \quad x^2 - y^2 + (x - y) = 0$

$\Rightarrow \quad (x-y)(x+y) + (x-y) = 0$

$\Rightarrow \quad (x-y)(x+y+1) = 0$

$\Rightarrow \quad x - y = 0 \quad [\because x + y + 1 \ne 0]$

$\Rightarrow \quad x = y$

$\therefore$ $f : N \to N$ is one-one.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

TR!CK

A function is an onto function, if its range is equal to its co-domain.

f is not onto because $x^2 + x + 1 \geq 3, \forall \ x \in N$
and so, 1, 2 does not have their pre-images.

Hence proved.

Short Answer Type-II Questions

1. Let A = set of real numbers

and $R = \{(a, b) : a \leq b^3\}$

(i) R is not reflexive, if $a = \frac{1}{2}$ and $b^3 = a^3 = \frac{1}{8}$

So, $\frac{1}{2}$ is not less than $\frac{1}{8}$.

(ii) R is not symmetric if $a \leq b^3$ then b is not less or equal to a^3, as $a = 1, b = 2, 1 < 2^3$ but 2 is not less than 1^3.

(iii) R is not transitive, if $a \leq b^3$, $b \leq c^3$ then it is not necessary that $a \leq c^3$.

Therefore, R is none of the reflexive, symmetric and transitive.

2. Let X is the set of sets and $A \in X, B \in X$ are two elements such that $A \subseteq B$.

Now $A \subseteq B$ implies that $B \subseteq A$ is not always true.

For example, if $A = \{1, 2\}$ and $B = \{1, 2, 3\}$ then $A \subseteq B$ but $B \not\subseteq A$.

In fact, $A \subseteq B \Rightarrow B \subseteq A$ is true only when $A = B$.

Therefore, relation '$\subseteq$' is antisymmetric.

Hence proved.

3. Here $f : R \to R$,

Given, $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

$f(x) = 1$ when $x > 0$, $f(1) = f(2) = 1$.

We see that the f image of 1 and 2 are 1.

$\therefore$ f is not one-one. ...(1)

Only 1, 0, −1 are the images, the other numbers are not the images of any element of the Domain.

$\therefore$ f is not onto. ...(2)

From eqs. (1) and (2),

f is neither one-one nor onto. **Hence proved.**

4. Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

then $f(x_1) = f(x_2)$

$\Rightarrow \cos x_1 = \cos x_2$

$\Rightarrow x_1 = 2n\pi \pm x_2$

$\Rightarrow x_1 \neq x_2$

$\therefore$ f is not one-one.

Also, $-1 \leq \cos x \leq 1$

$\therefore$ The numbers smaller than −1 and greater than 1 are in co-domain R which are not the image of any element of domain R.

$\therefore$ f is not onto. So, f is neither one-one nor onto.

Hence proved.

5. Here, function $f : R_+ \to [-5, \infty)$ is given as
$f(x) = 9x^2 + 6x - 5$.

TR!CK

f is one-one iff $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall \ x_1, x_2 \in X$
or f is one-one iff
$f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2 \ \forall \ x_1, x_2 \in X$

Let $x_1, x_2 \in R_+$ such that

$$f(x_1) = f(x_2)$$

then, $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow 9(x_1 + x_2)(x_1 - x_2) + 6(x_1 - x_2) = 0$

$\Rightarrow (x_1 - x_2)\{9(x_1 + x_2) + 6\} = 0$

$\Rightarrow x_1 - x_2 = 0$

$[\because x_1, x_2 \in R_+ \therefore 9(x_1 + x_2) + 6 \neq 0]$

$\Rightarrow x_1 = x_2, \forall \ x_1, x_2 \in R_+$

Therefore, $f(x)$ is one-one function.

TR!CK

A function is an onto function, if its range is equal to its co-domain.

Let y be any arbitrary element of $[-5, \infty)$

Then $y = f(x) \Rightarrow y = 9x^2 + 6x - 5$

$\Rightarrow y = (3x + 1)^2 - 1 - 5 = (3x + 1)^2 - 6$

$\Rightarrow (3x + 1)^2 = y + 6$

$\Rightarrow 3x + 1 = \sqrt{y + 6}$ as $y \geq -5 \Rightarrow y + 6 \geq 0$

$\Rightarrow x = \frac{\sqrt{y+6} - 1}{3}$

Therefore, f is onto, thereby range $f = [-5, \infty)$

Hence, $f(x)$ is a bijective function.

Long Answer Type Questions

1. Here, (a, b) and (c, d) relates by relation R when $ad = bc$ *i.e.*, first × fourth = second × third.

(i) Reflexive:

From the given condition,

(a, b) will be related to (a, b) by R when $a\,b = b\,a$

$\because$ For each $a, b \in N$, $a\,b = b\,a$ is true.

$\therefore (a, b) R (a, b) \Rightarrow$ Relation R is reflexive.

(ii) Symmetric:

Let $(a, b) R (c, d)$, then from given condition

$(a, b) R (c, d) \Rightarrow a\,d = b\,c \Rightarrow b\,c = a\,d$

$\Rightarrow c\,b = d\,a$ [from commutative law]

$\Rightarrow (c, d) R (a, b)$

$\because (a, b) R (c, d) \Rightarrow (c, d) R (a, b)$

$\therefore$ Relation R is symmetric.

(iii) Transitive:

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$

then from given condition,

$(a, b) R (c, d)$ and $(c, d) R (e, f)$

$\Rightarrow a\,d = b\,c$ and $c\,f = d\,e$

$\Rightarrow (a\,d) \times (c\,f) = (b\,c) \times (d\,e)$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$\Rightarrow \quad af = be$

$\Rightarrow (a, b)\, R\, (e, f)$

$\because \quad (a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$

$\Rightarrow \quad (a, b)\, R\, (e, f)$

$\therefore$ Relation R is transitive.

$\because$ The relation R on $N \times N$ is reflexive, symmetric and transitive.

Therefore, the relation R is equivalence relation on $N \times N$. **Hence proved.**

Let the equivalence class of $(2, 6)$ be (x, y)

Then

$$2y = 6x$$

This is possible when

$x = 1, y = 3$

$x = 2, y = 6$

$x = 3, y = 9$

Hence, the equivalence class of (2, 6) are (1, 3), (2, 6), (3, 9)....... $(n, 3n)$.

2. $N \times N = \{(a, b) : a \in N, b \in N\}$

Here the pair (a, b) related to (c, d) by R where $ad(b + c) = bc(a + d)$

i.e., first $\times$ fourth (second + third)

= second $\times$ third (first + fourth)

(i) Reflexive:

From the given condition.

(a, b) will be related to (a, b) by R.

when $\quad ab(b + a) = ba(a + b)$

or $\quad ab(a + b) = ab(a + b) \forall a, b \in N$

($\because$ Natural numbers are commutative with respect to addition and multiplication)

$\therefore \quad (a, b)\, R\, (a, b)$

$\Rightarrow$ Relation R is reflexive.

(ii) Symmetric:

Let $(a, b)\, R\, (c, d)$, then from given condition

$(a, b)\, R\, (c, d) \Rightarrow ad(b + c) = bc(a + d)$

$\Rightarrow bc(a + d) = ad(b + c)$ [commutative law]

$\Rightarrow cb(d + a) = da(c + b)$ [commutative law]

$\Rightarrow (c, d)\, R\, (a, b)$

$\therefore (a, b)\, R\, (c, d) \Rightarrow (c, d)\, R\, (a, b)$

$\therefore$ Relation R is symmetric.

(iii) Transitive:

Let $(a, b)\, R\, (c, d)$ and $(c, d)\, R\, (e, f)$

From the given condition,

$ad(b + c) = bc(a + d)$

and $cf(d + e) = de(c + f)$

$\Rightarrow \quad adb - bca = bcd - adc$

and $cfd - dec = def - cfe$

$\Rightarrow \quad ab(d - c) = cd(b - a)$

and $cd(f - e) = ef(d - c)$

$\Rightarrow \dfrac{ab}{b-a} = \dfrac{cd}{d-c}$ and $\dfrac{cd}{d-c} = \dfrac{ef}{f-e}$

$\Rightarrow \dfrac{ab}{b-a} = \dfrac{ef}{f-e}$

$\Rightarrow ab(f - e) = ef(b - a)$

$\Rightarrow abf + efa = abe + efb$

$\Rightarrow af(b + e) = be(a + f)$

$\Rightarrow (a, b)R(e, f)$

$\therefore (a, b)R(c, d)$ and $(c, d)R(e, f) \Rightarrow (a, b)R(e, f)$

$\therefore R$ is transitive

We see that the relation R on $N \times N$ is reflexive, symmetric and transitive.

Hence, R is an equivalence relation.

Hence Proved.

3. Since R and S are equivalence relations in set A so from the definition of the relation,

$R \subseteq A \times A$ and $S \subseteq A \times A$

$\Rightarrow R \cap S \subseteq A \times A$

$\Rightarrow R \cap S$ is a relation in set A.

Now, we will prove that $R \cap S$ is an equivalence relation.

$\because$ The relations R and S are reflexive.

$\therefore \quad (a, a) \in R$ and $(a, a) \in S \; \forall \; a \in A$

$\Rightarrow \quad (a, a) \in R \cap S \; \forall \; a \in A$

So, the relation $R \cap S$ is reflexive.

Let, $a, b \in A$ be such that $(a, b) \in R \cap S$

then, $\quad (a, b) \in R \cap S$

$\Rightarrow \quad (a, b) \in R$ and $(a, b) \in S$

$\Rightarrow \quad (b, a) \in R$ and $(b, a) \in S$

[$\because$ R and S are symmetric]

$\Rightarrow \quad (b, a) \in R \cap S$

So, the relation $R \cap S$ is symmetric.

Let, $\quad a, b, c \in A$ be such that,

$(a, b) \in R \cap S$ and $(b, c) \in R \cap S$

$\Rightarrow \quad [(a, b) \in R$ and $(a, b) \in S]$

and $\quad [(b, c) \in R$ and $(b, c) \in S]$

$\Rightarrow \quad [(a, b) \in R$ and $(b, c) \in R]$

and $\quad [(a, b) \in S$ and $(b, c) \in S]$

$\Rightarrow \quad [(a, c) \in R$ and $(a, c) \in S]$

[$\because$ R and S are transitive]

$\Rightarrow \quad (a, c) \in R \cap S$

So, the relation $R \cap S$ is transitive.

Thus, we see that the relation $R \cap S$ is reflexive, symmetric and transitive.

Hence, $R \cap S$ is an equivalence relation.

Hence proved.

4. Let $\quad A \in P(X)$.

Then, $\quad A \subset A \Rightarrow (A, A) \in R$

So, R is reflexive.

Let $\quad A, B, C \in P(X)$ such that

$(A, B), (B, C) \in R$

$\Rightarrow \quad A \subset B, B \subset C$

$\Rightarrow \quad A \subset C$

$\Rightarrow \quad (A, C) \in R$

So, R is transitive.

Let $\quad \phi, X \in P(X)$ such that $\phi \subset X$

Hence, $(\phi, X) \in R$. But, $X \not\subset \phi$ which implies that $(X, \phi) \notin R$.

Thus, R is not symmetric. **Hence proved.**

Get More Learning Materials Here : CLICK HERE www.studentbro.in

5. Given relation is $R = \{(a, b): a, b \in A, |a-b| \text{ is divisible by } 4\}$

and $A = \{x \in Z : 0 \le x \le 12\}$

Now, A can be written as

$$A = \{0, 1, 2, 3, \ldots 12\}$$

TiPS

- *A relation R defined an a set A is a reflexive, if $(x, x) \in R\ \forall x \in A$.*
- *A relation R defined on set A is a symmetric, if $(x, y) \in R \Rightarrow (y, x) \in R$ for any $x, y \in A$*
- *A relation R defined on a set A is a transitive, if $(x, y) \in R$ and $(y, z) \in R$.*
  $\Rightarrow$ *$(x, z) \in R$, for any $x, y, z \in A$.*

Reflexive: As for any $x \in A$, we get

$|x - x| = 0$, which is divisible by 4.

$\Rightarrow (x, x) \in R, \forall x \in A$

Therefore, R is reflexive.

Symmetric: As for any $(x, y) \in R$, we get $|x - y|$ is divisible by 4. (By using definition of given relation)

$\Rightarrow |x - y| = 4\lambda$, for some $\lambda \in Z$

$\Rightarrow |y - x| = 4\lambda$, for some $\lambda \in Z$

$\Rightarrow |y - x|$ is divisible by 4.

Thus, $(x, y) \in R \Rightarrow (y, x) \in R$, for any $x, y \in A$

Therefore, R is symmetric.

Transitive: For any, $(x, y) \in R$ and $(y, z) \in R$, we get $|x - y|$ is divisible by 4 and $|y - z|$ is divisible by 4. (By using definition of given relation).

$\Rightarrow |x - y| = 4\lambda$ and $|y - z| = 4\mu$

For some $\lambda, \mu \in Z$.

Now, $x - z = (x - y) + (y - z)$

$= \pm 4\lambda \pm 4\mu$

$= \pm 4(\lambda + \mu)$

$\Rightarrow |x - z|$ is divisible by 4.

$\Rightarrow (x, z) \in R$

Thus, $(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow (x, z) \in R$, for any $x, y, z \in A$

Therefore, R is transitive.

Since, R is reflexive, symmetric and transitive. So, it is an equivalence relation. **Hence proved.**

TR!CK

Let R be an equivalence relation in a set A and let $a \in A$. Then, the set of all those elements of A which are related to a under the relation R, is called the equivalence class determined by a.

Now, set of all elements related to 1 is {1, 5, 9}.

The set of all elements related to {2}.

$= \{a \in A : |2 - a| \text{ is divisible by } 4\} = \{2, 6, 10\}$

6. Given, a function $f : A \to B$, where $A = R - \{3\}$ and $B = R - \{1\}$, defined by $f(x) = \dfrac{x-2}{x-3}$

TiP

f is one-one iff $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in X$
or f is one-one iff
$f(x_1) \ne f(x_2) \Rightarrow x_1 \ne x_2$ for all $x_1, x_2 \in X$

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

Then, $\dfrac{x_1 - 2}{x_1 - 3} = \dfrac{x_2 - 2}{x_2 - 3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$

$\Rightarrow -3(x_1 - x_2) + 2(x_1 - x_2) = 0$

$\Rightarrow -(x_1 - x_2) = 0$

$\Rightarrow x_1 - x_2 = 0 \quad \Rightarrow x_1 = x_2$

Thus, $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2\ \forall\ x_1, x_2 \in A$

So, $f(x)$ is a one-one function.

Let $y \in B = R - \{1\}$ be any arbitrary element.

Then, $f(x) = y$

$\Rightarrow \dfrac{x-2}{x-3} = y$

$\Rightarrow x - 2 = xy - 3y$

$\Rightarrow x - xy = 2 - 3y$

$\Rightarrow x(1 - y) = 2 - 3y$

$\Rightarrow x = \dfrac{2 - 3y}{1 - y}$ or $\dfrac{3y - 2}{y - 1}$ ...(1)

TR!CK

A function is an onto function, if its range is equal to co-domain.

Clearly, $x = \dfrac{3y-2}{y-1}$ is a real number for all $y \ne 1$

Also, $\dfrac{3y-2}{y-1} \ne 3$

$\left[\because \dfrac{3y-2}{y-1} = 3 \Rightarrow 3y - 2 = 3y - 3 \Rightarrow 2 = 3, \text{ which is absurd}\right]$

Thus, for each $y \in B$, there exist $x = \dfrac{3y-2}{y-1} \in A$

such that $f(x) = f\left(\dfrac{3y-2}{y-1}\right) = \dfrac{\left(\dfrac{3y-2}{y-1}\right) - 2}{\left(\dfrac{3y-2}{y-1}\right) - 3}$

$= \dfrac{3y - 2 - 2y + 2}{3y - 2 - 3y + 3} = y$

TR!CK

A function $f : X \to Y$ is a bijective function, if it is both one-one and onto.

Therefore, $f(x)$ is an onto function.

Hence, $f(x)$ is a bijective function.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

7. Given, $f : R - \left\{-\frac{4}{3}\right\} \to R$ defined by

$$f(x) = \frac{4x}{3x+4}$$

Let $\quad x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$

TiP

f is one-one iff $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in X$
or f is one-one iff
$f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$ for all $x_1, x_2 \in X$

such that $\quad f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$$

$$\Rightarrow \quad (4x_1)(3x_2+4) = (3x_1+4)(4x_2)$$

$$\Rightarrow \quad 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$$

$$\Rightarrow \quad 16x_1 = 16x_2 \quad \Rightarrow x_1 = x_2$$

$\Rightarrow$ f is one-one.

Let $y \in R$.

The function f is onto if there exist

$x \in R - \left\{-\frac{4}{3}\right\}$ such that $f(x) = y$

Now, $\quad f(x) = y$

$$\Rightarrow \quad \frac{4x}{3x+4} = y$$

TR!CK

A function is an onto function, if its range is equal to co-domain.

$$\Rightarrow \quad 4x = 3xy + 4y$$

$$\Rightarrow \quad 4x - 3xy = 4y$$

$$\Rightarrow \quad x(4-3y) = 4y$$

$$\Rightarrow \quad x = \frac{4y}{4-3y} \in R - \left\{-\frac{4}{3}\right\}$$

Here we see that for $y = \frac{4}{3}$, x is not defined.

Thus, f is not onto.

8. We have, a function $f : R \to R$ defined by

$$f(x) = \frac{x}{x^2+1}, \forall\ x \in R$$

TiP

f is one-one iff $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in X$
or f is one-one iff
$f(x_1) \neq f(x_2) \Rightarrow x_1 \neq x_2$ for all $x_1, x_2 \in X$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$$\Rightarrow \quad \frac{x_1}{x_1^2+1} = \frac{x_2}{x_2^2+1}$$

$$\Rightarrow \quad x_1x_2^2 + x_1 = x_2x_1^2 + x_2$$

$$\Rightarrow \quad x_1x_2^2 - x_2x_1^2 + x_1 - x_2 = 0$$

$$\Rightarrow \quad x_1x_2(x_2 - x_1) - (x_2 - x_1) = 0$$

$$\Rightarrow \quad (x_2 - x_1)(x_1x_2 - 1) = 0$$

$$\Rightarrow \quad x_2 - x_1 = 0 \quad \text{or} \quad x_1x_2 - 1 = 0$$

$$\Rightarrow \quad x_1 = x_2 \quad \text{or} \quad x_1 = \frac{1}{x_2}$$

Here, f is not one-one as if we take $x_1 = \frac{1}{x_2}$

In particular, $x_1 = 2$ and $x_2 = \frac{1}{2}$, we get

$$f(2) = \frac{2}{4+1} = \frac{2}{5}$$

and $\quad f\left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{\frac{1}{4}+1} = \frac{1/2}{5/4} = \frac{2}{5}$

$$\because \quad f(2) = f\left(\frac{1}{2}\right),$$

but $\quad 2 \neq \frac{1}{2}$

So, f is not one-one.

TR!CK

A function is an onto function, if its range is equal to co-domain.

Let $y \in R$ (co-domain) be any arbitrary element.

Consider, $\quad y = f(x)$

$$\therefore \quad y = \frac{x}{x^2+1}$$

$$\Rightarrow \quad x^2y + y = x$$

$$\Rightarrow \quad x^2y - x + y = 0$$

$$\Rightarrow \quad x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$$

which does not exist for $1 - 4y^2 < 0$, *i.e.*, for $y > \frac{1}{2}$ and $y < -\frac{1}{2}$.

In particular for $y = 1 \in R$ (co-domain), there does not exist any $x \in R$ (domain) such that $f(x) = y$.

$\therefore$ f is not onto.

Hence, f is neither one-one nor onto. **Hence proved.**

9. Let $x, y \in R$ and $f(x) = f(y)$

$$\Rightarrow \quad \frac{x}{1+|x|} = \frac{y}{1+|y|}$$

If $x > 0$ and $y < 0$ then $x - y > 0$ and $2xy < 0$

$$\therefore \quad \frac{x}{1+x} = \frac{y}{1-y}$$

$$\Rightarrow \quad y + xy = x - xy$$

$\Rightarrow \quad 2xy = x - y$ which is not possible.

If x and y, both are positive then

$$f(x) = f(y)$$

$$\Rightarrow \quad \frac{x}{1+x} = \frac{y}{1+y}$$

$$\Rightarrow \quad x + xy = y + xy$$

$$\Rightarrow \quad x = y$$

If x and y both are negative then

$$f(x) = f(y)$$

Get More Learning Materials Here : CLICK HERE www.studentbro.in

$\Rightarrow \quad \frac{x}{1-x} = \frac{y}{1-y}$

$\Rightarrow \quad x - xy = y - xy$

$\Rightarrow \quad x = y$

$\therefore$ f is one-one.

$\because \quad y = \frac{x}{1+|x|}$

when $x \geq 0$,

$y = \frac{x}{1+x} \Rightarrow y + yx = x$

$\Rightarrow \quad x = \frac{y}{1-y}$

As $x \geq 0$,

$\therefore \quad \frac{y}{1-y} \geq 0 \Rightarrow \frac{y}{(1-y)} \leq 0$

$\Rightarrow \quad \frac{y(1-y)}{(1-y)^2} \leq 0 \Rightarrow y(1-y) \leq 0, y \neq 1$

$\Rightarrow \quad 0 \leq y \leq 1, y \neq 1 \Rightarrow 0 \leq y < 1$ ...(1)

when $x < 0$,

$y = \frac{x}{1-x} \Rightarrow y - yx = x$

$\Rightarrow \quad x(1+y) = y \quad \Rightarrow \quad x = \frac{y}{1+y}$

As $x < 0$,

$\frac{y}{1+y} < 0 \Rightarrow \frac{y(y+1)}{(y+1)^2} < 0$

$\Rightarrow \quad y(y+1) < 0 \Rightarrow -1 < y < 0$ ...(2)

From eqs. (1) and (2), we get

$-1 < y < 1$, which is equal to co-domain.

$\therefore$ f is onto.

Hence, f is one-one and onto. **Hence proved.**

Chapter Test

Multiple Choice Questions

Q 1. Let R be a relation on the set N of natural numbers defined by nRm if n divides m. Then R is:

a. reflexive and symmetric
b. transitive and symmetric
c. equivalence
d. reflexive, transitive but not symmetric

Q 2. Let $f : R \to R$ be defined by $f(x) = \frac{1}{x}$ $\forall$ $x \in R$.

Then f is:

a. one-one
b. onto
c. bijective
d. f is not defined

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false and Reason (R) is true

Q 3. Assertion (A): Let the relation R be defined in N by aRb, if $2a + 3b = 30$.

Then $R = \{(3, 8), (6, 6), (9, 4), (12, 2)\}$

Reason (R): An integer m is said to be related to another integer n, if m is a integral multiple of n. This relation in Z is reflexive, symmetric and transitive.

Q 4. Assertion (A): Let the function $f : R \to R$ be defined by $f(x) = 4x - 1$, $\forall$ $x \in R$. Then, f is one-one.

Reason (R): A function $f : X \to Y$ is defined to be one-one (or injective), if the images of distinct elements of X under f are distinct, *i.e.*,

$x_1, x_2 \in X$, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Case Study Based Questions

Q 5. Case Study 1

A relation R in a set A is called:

Reflexive: if $(a, a) \in R$, for every $a \in A$.

Symmetric: if $(a_1, a_2) \in R$, implies that $(a_2, a_1) \in R$, for any $a_1, a_2 \in A$

Transitive: if $(a_1, a_2) \in R$, $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$ for any $a_1, a_2, a_3 \in A$.

Based on the given information, solve the following questions:

(i) Show that the relation R in the set $A = \{1, 2, 3, ..., 13, 14\}$ defined as $R = \{(x, y) : 3x - y = 0\}$ is neither reflexive nor symmetric nor transitive.

(ii) Show that the relation R in the set Z of all integers defined as $R = \{(x, y) : x - y$ is an integer$\}$ is an equivalence relation.

(iii) Show that the relation R in the set A of human beings in a town $R = \{(x, y) : x$ is the father of $y\}$ is neither reflexive nor symmetric nor transitive.

Or

Show that the relation R in the set A of human beings in a town $R = \{(x, y)$: x is wife of $y\}$ is neither reflexive nor symmetric nor transitive.

Get More Learning Materials Here : CLICK HERE www.studentbro.in

Q 6. Case Study 2

An organisation conducted bike race under 2 different categories-boys and girls. Totally there were 250 participants. Among all of them finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B=\{b_1,b_2,b_3\}$ and $G=\{g_1,g_2\}$ where B represents the set of boys selected and G is the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions.

Based on the above information, solve the following questions:

(i) Let $R: B \to B$ be defined by $R=\{(x, y): x$ and y are students of same sex$\}$. Then show that the relation R is an equivalence relation.

(ii) Let $R: B \to G$ be defined by $R=\{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$, then the function R is many-one but onto.

(iii) Ravi wants to find the number of injective functions from B to G. How many numbers of injective functions are possible?

Very Short Answer Type Questions

Q 7. Let $A=\{0,1,2,3\}$ and define a relation R on A as follows: $R=\{(0,0),(0,1),(0,3),(1,0),(1,1),(2,2),(3,0),(3,3)\}$.

Is R reflexive, symmetric and transitive?

Q 8. If $A=\{a, b, c, d\}$ and the function $f=\{(a, b),(b, d),(c, a),(d, c)\}$, write f^{-1}.

Short Answer Type-I Questions

Q 9. If $A=\{1,2,3\}$ and f, g are relations corresponding to the subset of $A \times A$ as $f=\{(1,2),(2,1),(3,3)\}$ and $g=\{(1,2),(1,3),(2,1),(3,1)\}$, which of f, g is a function? Why?

Q 10. Let R be the equivalence relation in the set Z of integers given by $R=\{(a, b): 2$ divides $a-b\}$. Write the equivalence class $[0]$.

Short Answer Type-II Questions

Q 11. Is $g=\{(1,1),(2,3),(3,5),(4,7)\}$ a function? If g is described by $g(x)=\alpha x+\beta$, then what value should be assigned to α and β?

Q 12. Let the function $f: R \to R$ be defined by $f(x)=\cos x, \forall x \in R$. Show that f is neither one-one nor onto.

Long Answer Type Questions

Q 13. If $A=\{1,2,3,4\}$, define relations on A which have properties of being:

(i) reflexive, transitive but not symmetric.

(ii) symmetric but neither reflexive nor transitive.

(iii) reflexive, symmetric and transitive.

Q 14. Let $A=[-1,1]$, then discuss whether the following functions defined on A are one-one onto or bijective.

(i) $f(x)=\frac{x}{2}$

(ii) $g(x)=|x|$

(iii) $h(x)=x|x|$

(iv) $k(x)=x^2$

Get More Learning Materials Here : CLICK HERE www.studentbro.in